Matching with preferences: Theory and Practice

Suppose that a CS department assigns a adviser to each new graduate student. Each student has his/her own preference for professors and each professor does for students. The assignment should be fair to both students and professors, where the theory of stable matchings plays a key role. In this course, stable matchings, or more generally matchings under preferences, are studied mainly from a theoretical point of view but also on its wide range of applications to the real world. The Nobel Prize 2012 for Economics was awarded to Alvin E. Roth and Lloyd S. Shapley. Shapley, with David Gale, first gave the notion of stable matchings as well as the Gale-Shapley Algorithm and Roth has had a lot of contribution to the field both from Computer Science and Economics angles. The topic is strongly related to the main research area of the instructor.

1. Stable matchings, Introduction
A stable matching is a sort of bipartite matching. So we start with standard bipartite matchings and then introduce the notion of stable matchings, observing their basic ideas.

1.1 Maximum bipartite matchings
1.2 Maximum weighted bipartite matchings
1.3 Men-Women matchings; Preference lists
1.4 Blocking pairs
1.5 Removing blocking pairs

2. Stable marriage problem
As shown in the first chapter, obtaining a stable matching is not trivial at all. Thus the novelty of the Gale-Shapley algorithm should be highly appreciated. We study its several important features in this chapter.

2.1 Gale-Shapley algorithm; its novelty
2.2 Correctness and time complexity
2.3 Men/Women optimal stable matchings
2.4 Average and worst case complexity

3. Mathematical structures of stable matchings
For a given instance, i.e., a set of preference lists of men and women, there are a lot of different stable matchings in general. So we may want to obtain a "good" stable matching among them. For this purpose, it plays a key role that this set of all stable matchings has a beautiful mathematical structure, as studied in this chapter.

3.1 Finding stable matchings with specific features
3.1 Describing all stable matchings
3.2 Rotations

4. Some extensions to preference lists

If the number of men and women is large, it may not be realistic to provide a total order of all people of the different sex. Thus it is quite natural to allow incomplete lists and indifferences. Unfortunately we can no longer claim several features of original stable matchings given so far. The problem becomes much harder in some cases.

4.1 Incomplete lists
4.2 Indiffereces
4.3 Intractability and approximation

5. Hospital residence problem

One-to-one stable matchings so far discussed are definitely a beautiful mathematical model, but when considering applications, one-to-many versions are more popular. The hospital residence problem is widely known as the most important model for real world application.

5.1 Definitions and solutions
5.2 Rural hospital theorem
5.3 Quota lower bounds

6. Stable roommate problem

Here we extend the notion of stability from for bipartite matchings to for matchings in general graphs. Namely each of $2n$ people submits a preference list including all the other $2n-1$ people and our goal is to obtain a stable set of n pairs. We can use an algorithm similar to Gale-Shapley, but its design and analysis is much harder than the one-to-one case.

6.1 Stable roommate as stable marriage
6.2 How to obtain stable roommates

7. Real-world applications

A bit surprisingly, what was essentially the Gale-Shapley algorithm was already being used as early as 1952! by the National Intern Matching Program in the US. We study its rich history and a wide range of real-world applications of stable matchings in general.

7.1 Hospital residence matching programs in US, UK, and others
7.2 Student assignment
7.3 Kidney exchange

8. Other models for matching with preferences

Preference lists of agents provide several interesting notions other than stability. We see some of them and why they are interesting from an algorithmic point of view.

8.1 Weak and strong stabilities

8.2 Pareto optimality
8.3 Popular matchings
8.4 Online stable matchings

9. Game theoretical aspects of stable matchings
Researchers in Computer Science often look at problems from an algorithmic point of view, especially whether they are tractable or intractable. Researchers in Economics have several different angles. A typical one of them is a game-theoretic point of view. For instance, it is known at an early stage that the men-proposing Gale-Shapley algorithm is strategy-proof for men. We study this theorem as well as several others.