POLYHEDRAL COMBINATORICS (EXTENDED FORMULATIONS): EXERCISE SHEET 1

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Ex. 1: Let $M_1 \subset \mathbb{R}^{m_1 \times d}$ and $M_2 \subset \mathbb{R}^{m_1 \times d}$ be non-negative matrices. Define matrix $M \subset \mathbb{R}^{(m_1+m_2)\times d}$ obtained by concatenating M_1, M_2. That is

$$M = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}$$

Prove that $\text{cc}(M) \leqslant \max\{\text{cc}(M_1), \text{cc}(M_2)\} + 1$.

Ex. 2: Let P be a polytope and let Q be an extended formulation of P. Which of the following is true?

(1) $\text{xc}(P) \leqslant \text{xc}(Q)$
(2) $\text{xc}(P) = \text{xc}(Q)$
(3) $\text{xc}(P) \geqslant \text{xc}(Q)$

Why?

Ex. 3: Let P be a polytope and let F be a face of P. Prove that $\text{xc}(F) \leqslant \text{xc}(P)$.
(**Hint:** If $P \cap \{x \mid \alpha^T x = \beta\}$ is a face of P then what can you say about $Q \cap \{x \mid \alpha^T x = \beta\}$, where Q is an extended formulation of P?)