Exercises from Lecture 1

We formulated the asymetric travelling salesman problem as follows. Let n be the number of cities and a_{ij} be the distance from city i to city j, which is not necessarily the same as a_{ji}.

$$\max \sum_{1\le i,j\le n} a_{ij} x_{ij}$$

$$s.t. \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n \tag{2}$$

$$\sum_{i,j\in S} x_{ij} \le |S| - 1, \quad S \subset \{1, 2, \ldots, n\}, S \ne \emptyset \tag{3}$$

$$x_{ij} = 0/1 \quad 1 \le i, j \le n \tag{4}$$

1. Prove that feasible solutions to (1)-(4) are travelling salesman tours and vice versa.

2. Using lp_solve, try to find a small problem for which the fractional solution (replace (4) by $0 \le x_{ij} \le 1$) is strictly smaller than the integer solution.

3. Find an efficient method to solve the fractional knapsack problem:
$\max c^T x$
$s.t.\ a^T x \le b, \quad 0 \le x \le 1$
for integer vectors a, c of length n and integer constant b.

4. Find a natural combinatorial problem for which the feasible solutions are those that satisfy (1),(2),(4) above.