Energy-Optimal Online Algorithms for Broadcasting in Wireless Network

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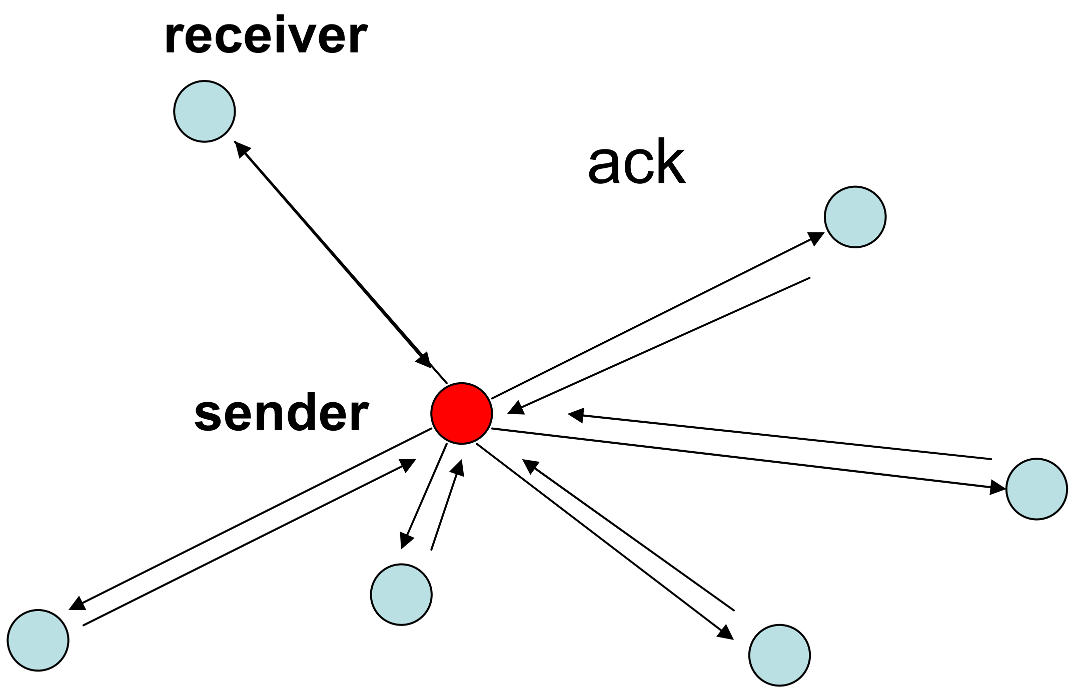
Outline

- Background
- Model
- Problems and Results
- Algorithms and Analyses
  - Single Receiver Case
  - Multiple Receivers Case
- Conclusion

Background

- Design energy-efficient online message broadcasting protocols in ad-hoc wireless networks
  - energy-efficiency :
    - save the battery-resource
  - online property: ･･･ ad-hoc network
    - non-static network

Problem



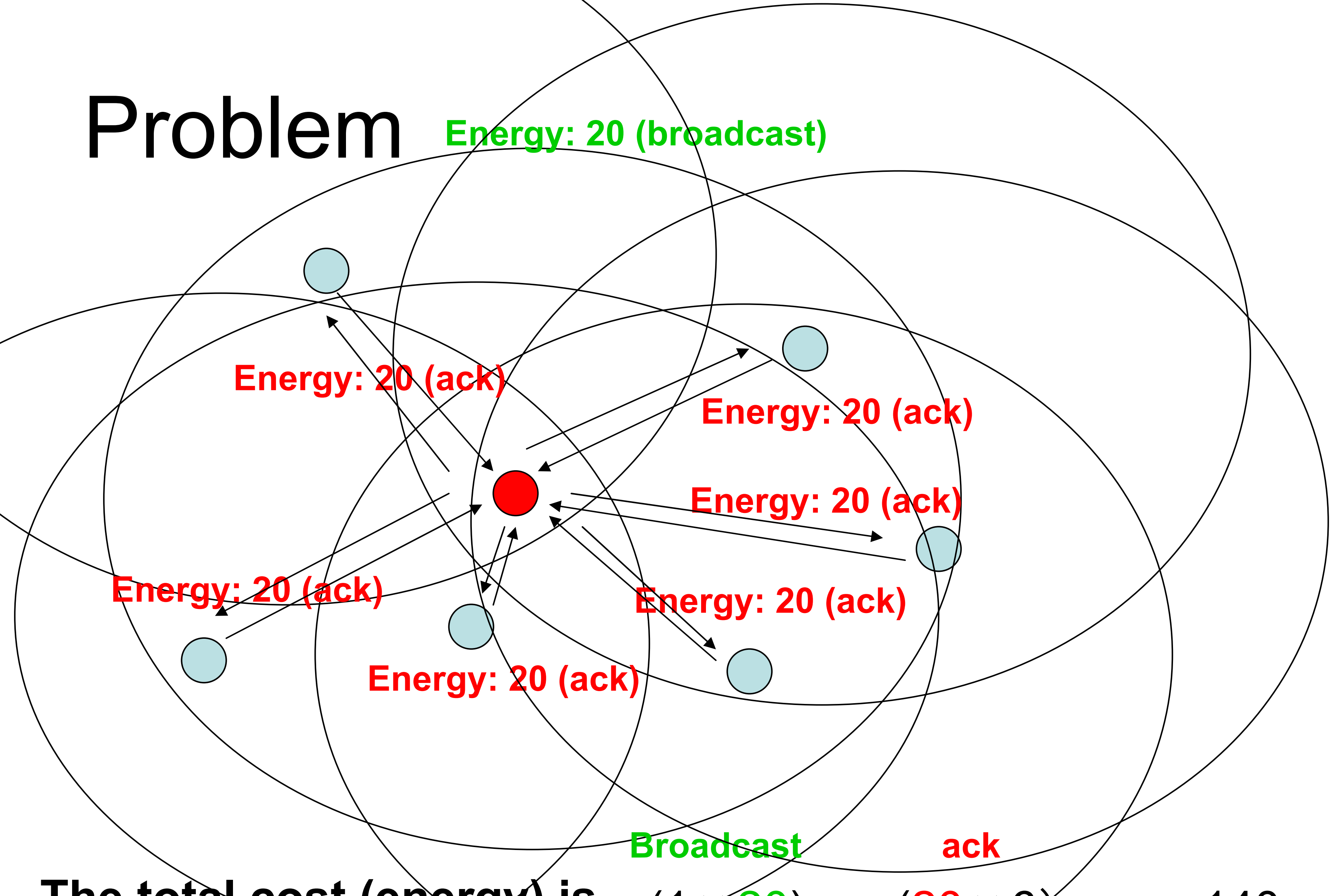
- The sender does not know the distances to receivers
- Broadcast a message to nearest *n-1* hosts
- Receiving ack from the *n-1* hosts

Problem: Design a good online algorithm

Problem

- Communication requires energy consumption.
- The energy consumption depends on
  the distance between the sender and receivers.
  (The distance is longer, the energy must be larger.)
- The **s**ender/**r**eceiver have no distance information.
- **s** sends some message (e.g., beacon) to **r.**
  with some energy consumption.
- If **r** receives the beacon, he needs to send "ack"
  to **s** with the same amount of energy consumption.

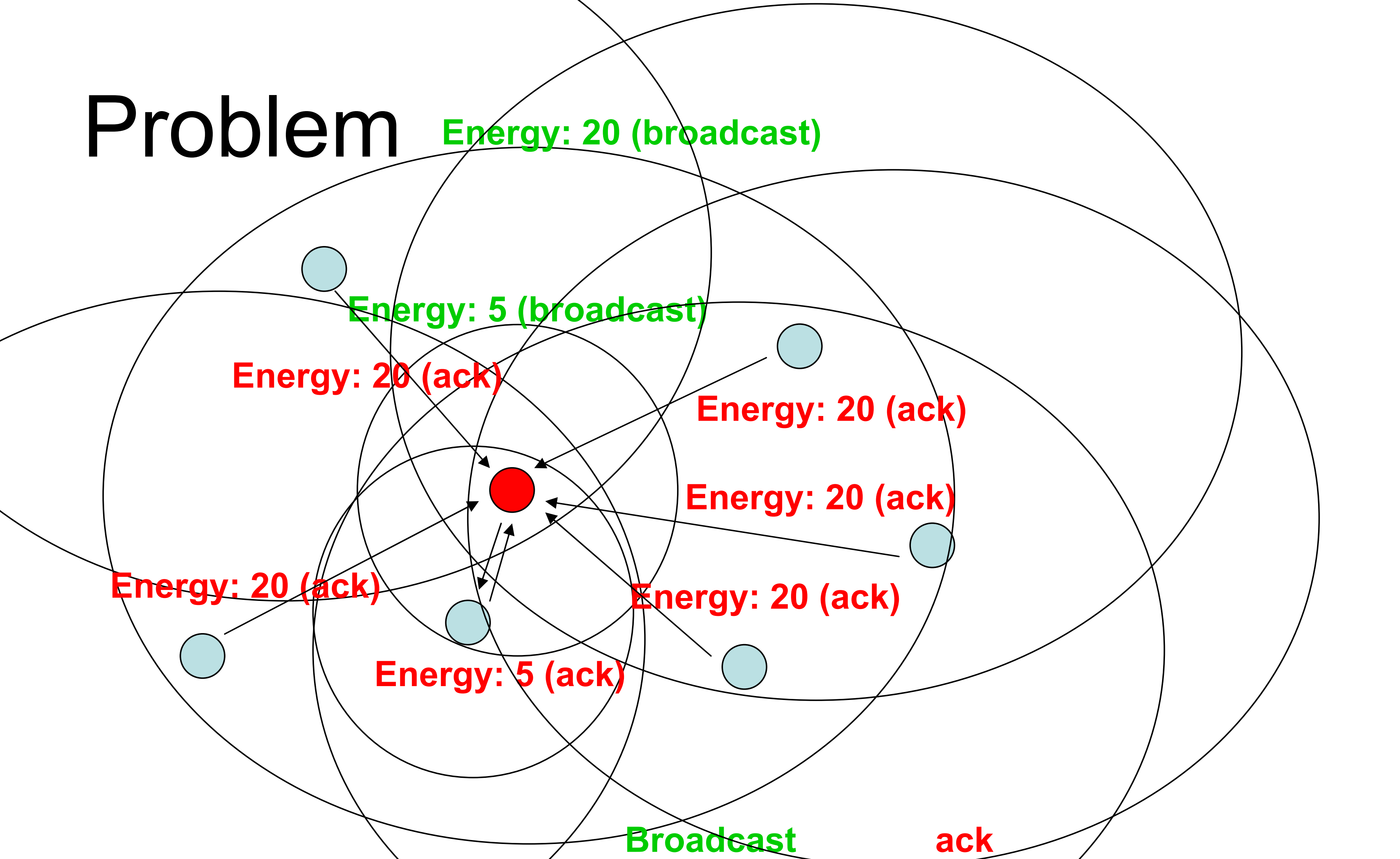
Problem



The total cost (energy) is $\underbrace{(1 \times 20)}_{\text{Broadcast}} + \underbrace{(20 \times 6)}_{\text{ack}} = 140$

Problem: Design a good online algorithm

Problem



Broadcast ack

The total cost (energy) is (5+20)+(5×1 +20×5)＝130 **＜140**

Problem: Design an energy-efficient algorithm good online algorithm

Model

- The attenuation of signal power P_s is $\tilde{P}_s = \frac{P_s}{d(s,t)^{\delta}}$
where $d(s,r)$ is the distance
between s and r, and $\delta \geqq 1$ is
the distance-power gradient.
γ is the minimum power to decode a message.
  – The maximum distance to which a message can derivered from s is $(P_s / \gamma)^{1/\delta}$

- Only a direct broadcast is allowed.
(No multi-hop delivery is allowed.)

- Synchronous Communication ･･･
(We can utilize a global clock and unique IDs of nodes)

- Collision-free and Failure-free

Related Works

- Range Assignment Problem
  - offline : The distances between any pair of hosts are given.
  - Minimizing the total energy consumption to broadcast a message to a set of recipients
  - Constructing energy-efficient multicast tree with several properties:
    - connectivity from a source, strong connectivity, small radius, and so on

"On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs", [CCPRV 2001]

"Power Consumption in Packet Radio Networks", [KKKP 2000]

Problems and Results

Algorithms and its Performance

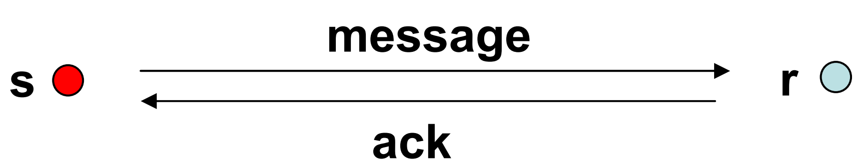
- Minimize the total energy consumption
- Our model is "online", i.e., no a-priori information.
- Use competitive analysis:
- The performance of algorithm A (competitive ratio) is

$$\sup\left\{ \frac{\text{cost}(A, I)}{\text{cost}^*(I)} \,\middle|\, I : \text{instance} \right\}$$

- cost* : the minimum value of the total energy consumption with complete information

Problems and Results (1)

- **Problem BA2** (Broadcast+Ack-2)
  - one sender *s* and one receiver *r*
  - s sends a message to *r*.
  - *r* sends an ack to s after receiving the message.



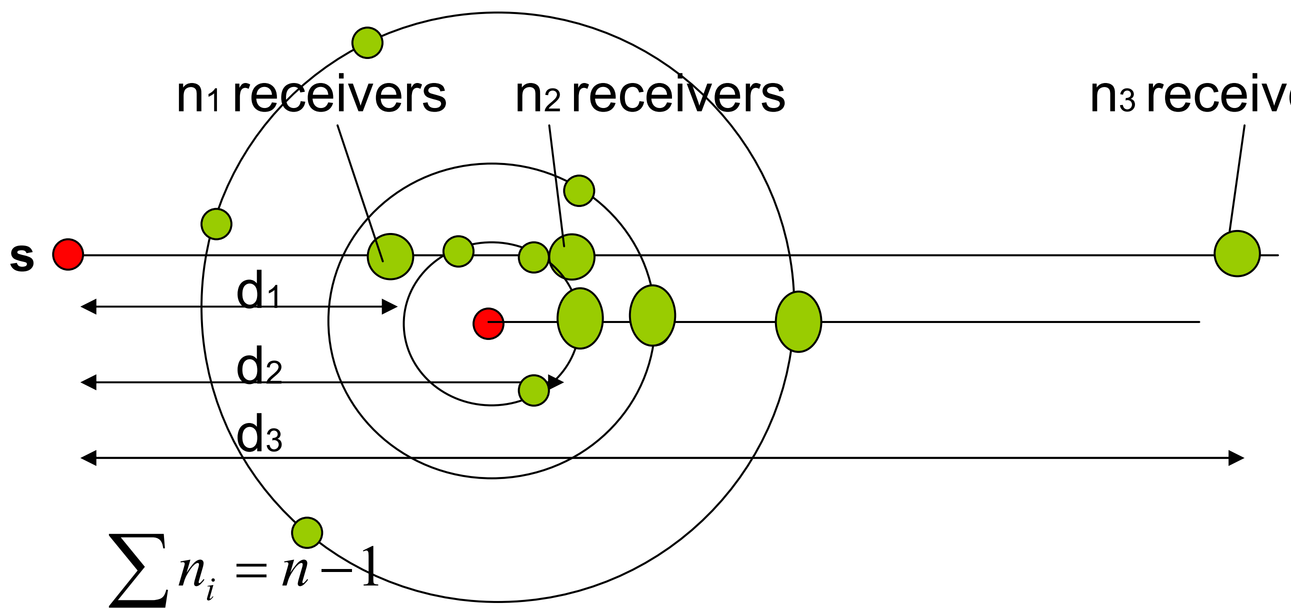
  - $\text{cost}^* = 2p^* = 2\gamma \cdot d^{\delta}$
- **Theorem**

  The optimal competitive ratio of problem BA2 is $3/2+\sqrt{2}$

(No online algorithm whose competitive ratio is smaller than $3/2+\sqrt{2}$ **)**

Problems and Results (2)

- **Problem BAn** (Broadcast+Ack-n)
  - one sender *s* and n-1 receivers, $r_{1,} r_2, \ldots, r_{n-1}$
  - s sends a message to $r_{1,} r_2, \ldots, r_{n-1}$.
  - Each r sends an ack to s after receiving the message.



$$\sum_{i=1,..,k} n_i = n - 1$$

Problems and Results (3)

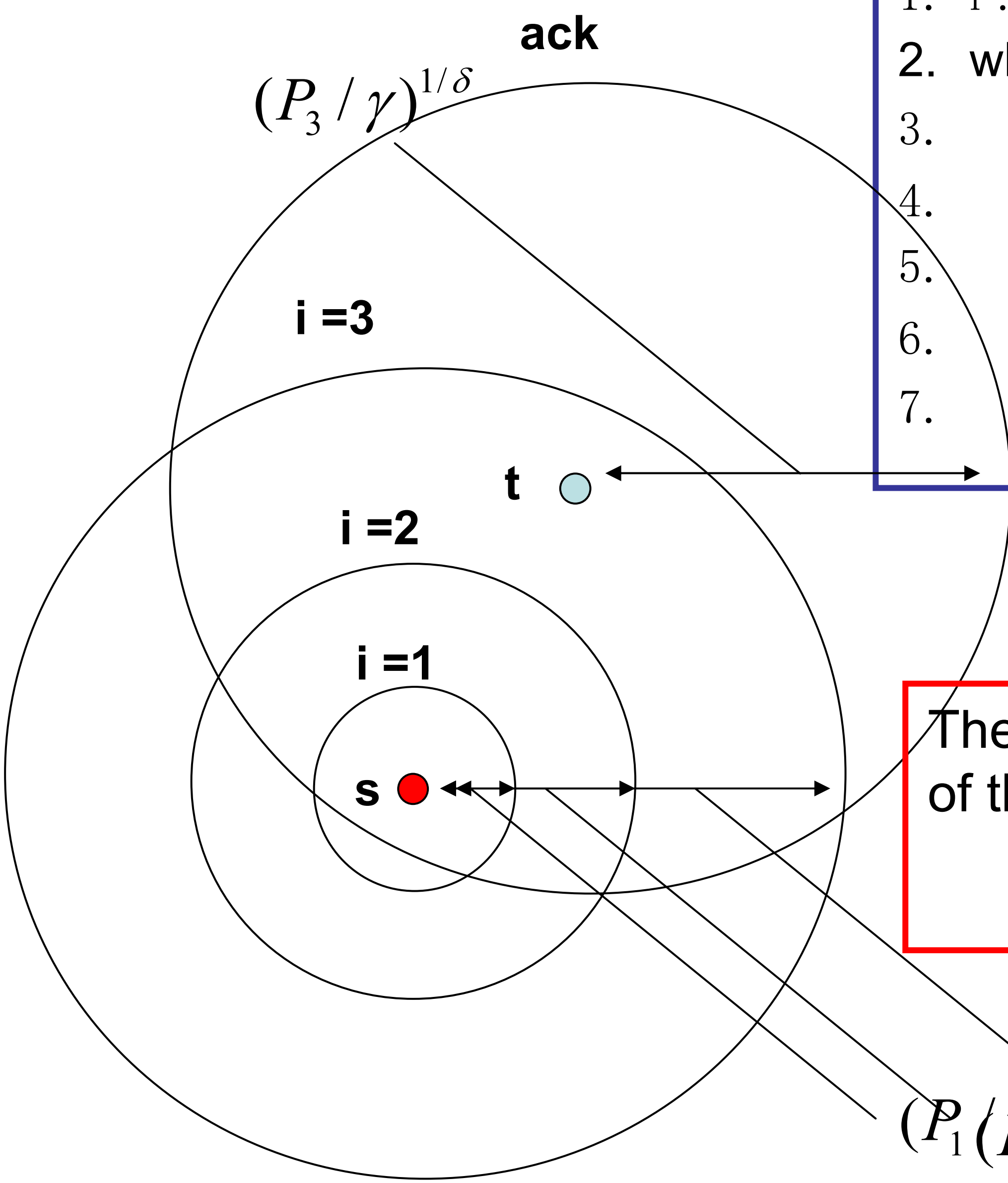
Theorem

The optimal competitive ratio of problem BAn is $3/2+\sqrt{2}$.

Algorithms and Analyses

Generic Protocol (Algorithm)

```
Procedure SendMessage(t,msg)
i := 1, f := true
while f
    do Transmit(msg,pi) with power pi.
        wait.
        if received ack from t
            f := false;
        i := i+1;
```




```
Procedure SendMessage(t,msg)
i := 1, f := true
while f
    do Transmit(msg,pi) with power pi.
        wait.
        if received ack from t
            f := false;
        i := i+1;
```

The total cost (energy consumption) of the procedure is

$$\mathrm{cost}(A) = \sum_{i=1}^{J} p_i + p_J$$

Single Receiver Case (1)

- **Algorithm: DA[β]** (Doubling Algorithm)
  - In SendMessage,

    Set $p_1 = \gamma$ and $p_{i+1} = \beta \cdot p_i$.

$$p_1 = \gamma$$

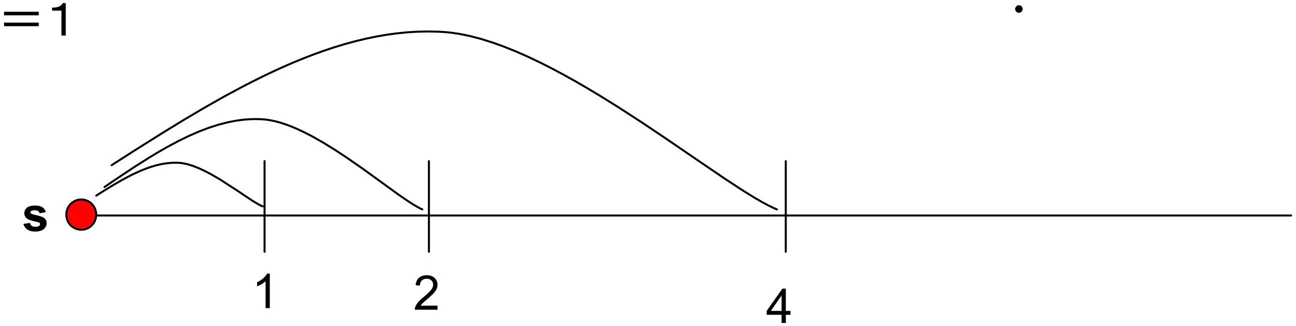
$$p_2 = \gamma \cdot \beta$$

$$p_3 = \gamma \cdot \beta^2$$

$$\vdots$$

$\beta = 2$

$\gamma = 1$



Single Receiver Case (2)

- **Proposition**:
  DA[β] algorithm achieves the competitive ratio is $\frac{\beta(2\beta-1)}{\beta-1}$ for problem BA2.

The value $\frac{\beta(2\beta-1)}{\beta-1}$ is minimized when $\beta = 1 + \frac{1}{\sqrt{2}}$,

and it is $\frac{3}{2} + \sqrt{2}$.

Single Receiver Case (3)

- Theorem
  The optimal competitive ratio of problem BA2 is $3/2+\sqrt{2}$

Sketch of Proof:

1. Assume an optimal online algorithm with competitive ratio c ≦ 3/2+√2 and its output, say x_1, x_2, x_3 ….

2. From the competitive ratio property, we have the following inequality,

$$x_{i+1} \le \alpha_i \cdot x_i,\ \alpha_i = c + \frac{1}{2} - \frac{c}{\alpha_{i-1}}$$

Sketch of Proof: (continued)

3. The parameter sequences α_i is a Cauchy sequence, so α_i converges to a real value α, and we have the following quadratic equation:

$$\alpha = c + \frac{1}{2} - \frac{c}{\alpha}$$

4. From the condition of the existence of α is

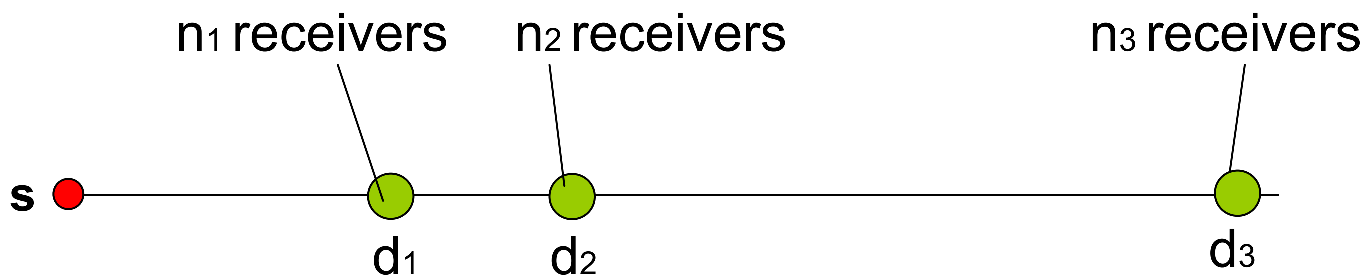
$$c \geq \frac{3}{2} + \sqrt{2}$$

5. The algorithm achieves the competitive ratio; i.e., it gives the upper bound.

Q.E.D

Multiple Receivers Case(0)

- **Problem BAn** (Broadcast+Ack-n)
  - one sender *s* and n-1 receivers, $r_1, r_2, \ldots, r_{n-1}$
  - s sends a message to $r_1, r_2, \ldots, r_{n-1}$.
  - Each r sends an ack to s after receiving the message.

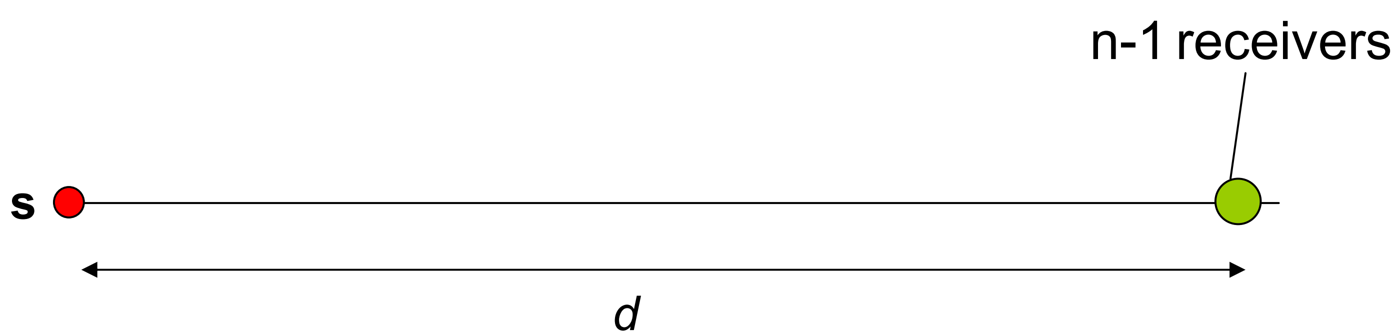


- Offline case:
  A simple Dynamic Programming can solve this in liner time.

Multiple Receivers Case (1)

Consider a simple special case:

- **Problem UBAn** (Broadcast+Ack-n)
  - one sender *s* and n-1 receivers, $r_1, r_2, \ldots, r_{n-1}$ all at the same distance d from s.
  - s sends a message to $r_1, r_2, \ldots, r_{n-1}$.
  - Each r sends an ack to s after receiving the message.



Multiple Receivers Case (2)

- **Proposition**

The competitive ratio of DA[β] algorithm fixing $\beta = 1 + \frac{1}{\sqrt{n}}$, for UBAn is at most $1 + \frac{2}{\sqrt{n}} + \frac{1}{n}$

- **Theorem**

The optimal competitive ratio of problem UBAn is $1 + \frac{2}{\sqrt{n}} + \frac{1}{n}$

The previous proofs can be extended to this case.

Dynamic Doubling Algorithm

To solve the general case, we propose the following online algorithm:

Procedure DDA(n,msg)

1. p:= γ
2. **while** n>1
3. **do** Transmit(msg,p) with power p.
4. wait.
5. n:=n−#(received ack)
6. p:= p· β_n

$$\beta_k = 1 + \frac{1}{\sqrt{k}}$$

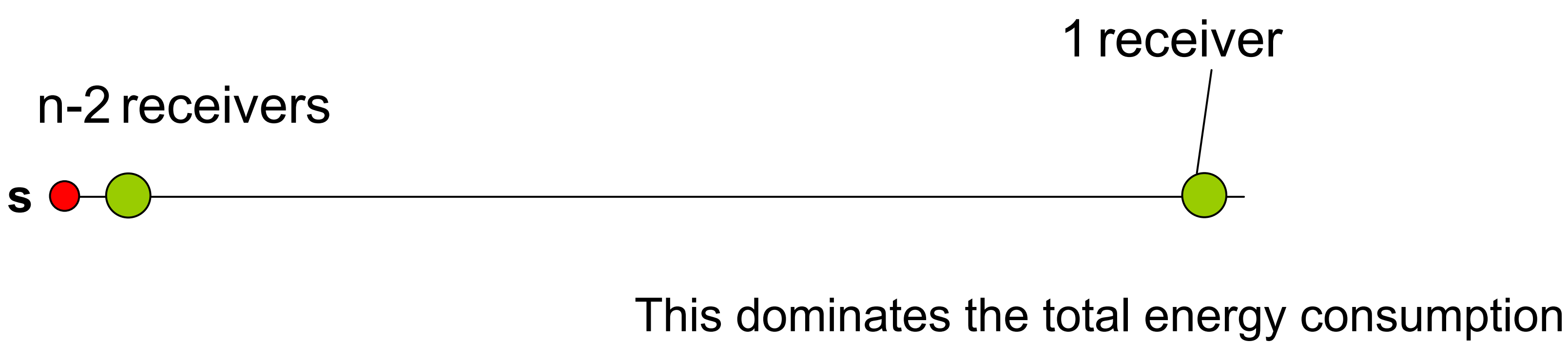
Results of Multiple Receivers (1)

Theorem

The optimal competitive ratio of problem BAn is $3/2+\sqrt{2}$.

Sketch of Proof:

- Lower bound : Consider the following situation (instance):



Sketch of Proof:

- Upper bound : DDA algorithm achieves the competitive ratio $3/2+\sqrt{2}$.

  1. The problem instance can be considered the union of $UBAn_1$ and $BA(n-n_1)$.

  2. In the $UBAn_1$ part, DDA algorithm achieves competitive ratio $1+\frac{2}{\sqrt{n_1}}+\frac{1}{n_1}\leq\frac{3}{2}+\sqrt{2}$

  3. By applying this discussion repeatedly, the competitive ratio of each part is at most $3/2+\sqrt{2}$, so in total the competitive ratio is $3/2+\sqrt{2}$.

Q.E.D

Conclusion

- Direct broadcast on online setting
- Single receiver and multiple receivers
- Energy-optimal online algorithms
  - doubling algorithm and
    dynamic doubling algorithm
  - The optimal competitive ratios are
    both $3/2+\sqrt{2}$

Future Work

- Not only energy-efficient
  but also time-efficient online algorithm
- Considering failure, collision, and so on