

**POLYHEDRAL COMBINATORICS (EXTENDED FORMULATIONS):  
EXERCISE SHEET 2**

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**Ex. 1:** Let  $P, K$  and  $Q$  be three polytopes in  $\mathbb{R}^d$  such that  $P \supseteq K \supseteq Q$ . Define the slack matrix of  $P$  with respect to  $Q$ , denoted by  $S(P, Q)$ , as follows: the  $(i, j)$ -th entry contains the slack of the  $i$ -th facet of  $P$  with the  $j$ -th vertex of  $Q$ . Let  $S(K)$  be slack matrix of  $K$ . Then prove that

$$\text{cc}(S(K)) \geq \text{cc}(S(P, Q))$$

where  $\text{cc}(M)$  denotes the communication complexity of matrix  $M$ .

**(Hint:** Construct a protocol for  $S(P, Q)$  from that of  $S(K)$ . Use facts like: vertices of  $Q$  can be represented as a convex combination of vertices of  $K$ .

**Ex. 2:** Let  $P_1, P_2$  be two polytopes in  $\mathbb{R}^d$ . Give upper bounds on the extension complexity of the following polytopes in terms of that of  $P_1$  and  $P_2$ .

- $P_1 \cap P_2$
- $\text{conv}(P_1 \cup P_2)$ , where  $\text{conv}(S)$  denotes the convex hull of  $S$ .